Power Requirements and Blend Times in the Agitation of Pseudoplastic Fluids

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Agitation and blending studies were made with solutions of Natrosol 250H, which were pseudoplastic fluids with nonconstant flow-behavior indexes. The published power number-Reynolds number correlation for pseudoplastic fluids having constant flow-behavior indexes is valid for Natrosol solutions. Published correlations for blend times in Newtonian fluids do not apply to Natrosol solutions; the blend times are ten to fifty times longer than with Newtonian fluids of the same apparent viscosity. Blend times are shortest in unbaffled vessels when a vortex is formed and correlate well with the vortex depths. Insertion of radial baffles always lengthened the blend time.

Agitation and mixing are basic steps in many chemical processes. Although many of the fluids handled are non-Newtonian, it has only been recently that the literature has dealt with the mixing and blending of these fluids.

There are two general areas of interest: first the dependency of power requirements on vessel and impeller geometry and fluid properties and second the time required to reach a certain degree of homogeneity in a blending operation. Metzner and Otto (5) proposed a relationship between shear rate and impeller rotational speed which made it possible to define an apparent viscosity for a non-Newtonian fluid and therefore a modified Reynolds number. Metzner and coworkers (4) then applied this modified Reynolds number to the power requirements on vessel and impeller tonian fluids by means of the familiar power number–Reynolds number correlation developed for Newtonian fluids. Their work was concerned with fluids having constant flow-behavior indexes.

The literature contains no papers that deal with blend times for non-Newtonian fluids, though Norwood and Metzner (7) have suggested that their correlation for Newtonian fluids could be used with pseudoplastic fluids. They fail to indicate however how the apparent viscosity of the pseudoplastic fluid should be calculated for this purpose.

The purposes of this investigation were to apply the power number–Reynolds number correlation to pseudoplastic fluids with varying flow-behavior indexes, to establish a relationship for blending rates for these fluids, and to propose a general criterion for scale up.

APPARATUS

The mixing equipment used in this investigation consisted of a ¼-hp. motor with variable-speed drive, a torque table and pan balance to measure the power input to the fluid, three glass mixing vessels, and three flat-bladed turbines.

The mixing vessels were Pyrex brand flat-bottom cylindrical jars, 5.7, 11.4, and 17.3 in. I.D. The two smaller vessels were fitted with four removable baffles having a width one-tenth of the tank diameter. The baffles were set ¼ in. out from the wall of the vessels so that fluid could circulate behind them.

Three standard flat-bladed turbines with diameters of 2, 4, and 6 in. were used. Each turbine had six blades. Geometric similarity was maintained in all runs with a ratio of vessel diameter to impeller diameter of 3:1. The depth of fluid in the vessel was equal to the vessel diameter.

For this investigation a non-Newtonian fluid was desired, the viscosity of which could be easily varied by the addition of a suitable solute. Because of the method used for determining blend time, the viscosity could not be affected by the presence of moderate concentrations of acids, bases, or salts The material chosen was Natrosol 250H, a high-molecular-weight cellulose derivative, hydroxyethyl cellulose, which is readily soluble in water and forms a slightly cloudy solution. Because of the high molecular weight only a small percentage by weight is needed to obtain a viscous solution (25,000 centipoise for a 2% solution). The density of the solution is that of water. Natrosol exhibits no time-dependent properties at the concentrations studied (up to 2% by weight) and does not degrade when kept for several months. Its viscosity is not affected by moderate concentrations of acids, bases, or salts.

Natrosol is a pseudoplastic fluid that does not follow the simple power-law equation

$$\tau = K\frac{(dv)^{n'}}{(dr)} \qquad (1)$$

where K and n' are constants. Instead the flow-behavior index n' varies with shear rate.

Figure 1 shows the relationship between shear stress and shear rate for several concentrations of Natrosol at 25°C. Curve D is for a 1.0% solution of Natrosol. For shear rates below about 10 sec.$^{-1}$ the curves were established with a Brookfield synchroelectric viscometer, calibrated with Newtonian fluids of known viscosity. Shear stress was found from the product

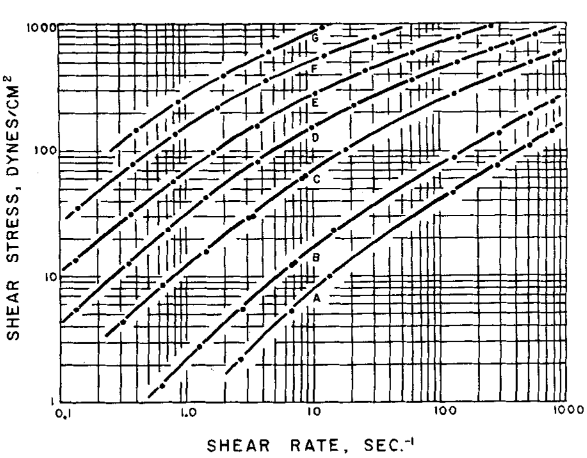

Fig. 1. Flow curves for various concentrations of Natrosol.

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of a constant and the instrument scale reading; shear rate was found from a constant and the slope of a logarithmic plot of scale reading against rotational speed, following the method of Kreiger and Maron (*3*). At higher shear rates a calibrated Texaco viscosimeter, type B, was used. This instrument gives directly the shear stress at the stationary cup surface; it was converted to the shear stress at the bob surface through the ratio of the cup diameter to bob diameter. The data from the two instruments agreed well. The apparent viscosity at any shear rate was found from Figure 1 by dividing shear stress by shear rate.

In the study of blend time a saturated solution of phenolphthalein was added to the fluid. Acid was introduced and neutralized with base. The completion of a run was visually observed by the color change of phenolphthalein, which occurs at a pH of 9.

The ranges of variables studied are given in Table 1.

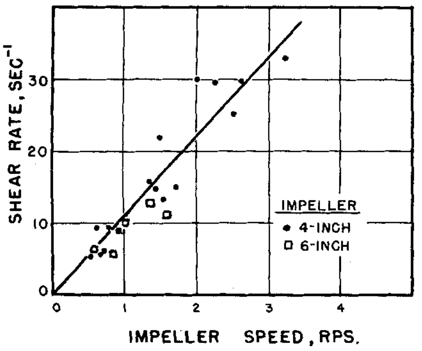

Fig. 2. Average shear rate for various impeller rotational speeds.

RESULTS—POWER REQUIREMENTS

Metzner and Otto (*5*) proposed that the average shear rate in an agitated vessel varies linearly with the rotational speed of the impeller, according to the equation

$$\left(\frac{dv}{dr}\right)_{avg} = kN \qquad (2)$$

They state that for Equation (2) to be valid the flow-behavior index *n'* in Equation (1) should be constant. In testing this relationship they chose fluids that had constant *n'* values for the range of shear rates studied. Other investigators (*1, 4*) testing this relationship also used fluids with constant *n'* values. The fluids used in this investigation had varying *n'* values over the range of shear rates investigated.

The values of *k* in Equation (2) were computed with results obtained with the 4- and 6-in. impellers. Since runs only in the laminar region are useful for determining *k*, the range of shear rates was limited. The value of *n'* varied from 0.6 to 0.3 as the rotational speed increased.

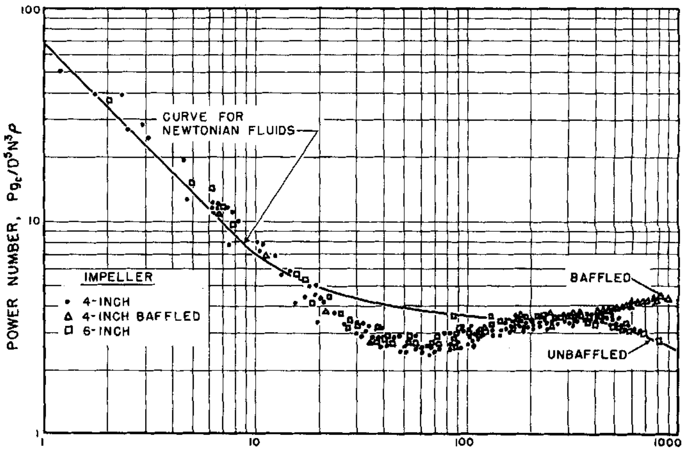

Fig. 3. Power number–Reynolds number correlation for pseudoplastic fluids.

Figure 2 is a plot of average shear rate vs. rotational speed for a variety of fluids having different values of *n'*. The points scatter somewhat but fairly closely define a straight line with a slope of about 11. The results shown in Figure 2 indicate that Equation (2) is more general than originally proposed since it also holds for fluids that do not have a constant flow-behavior index.

Metzner and Otto originally suggested that $k = 13$, but recently Metzner and co-workers used $k = 11$, and Calderbank and Moo-Young used $k = 10$. A value of $k = 11$ was used throughout this investigation to compute the apparent viscosity of the fluids.

The power required is plotted vs. Reynolds number in Figure 3, showing data obtained with the 4- and 6-in. impellers. Both baffled and unbaffled systems were studied with the 4-in. impeller. All runs with the 6-in. impeller were with unbaffled vessels. The dotted line is the curve for Newtonian fluids as reported by Rushton et al. (*8*). The curves are identical for the laminar region ($N_{Re} < 10$). The laminar region for pseudoplastics is extended to $N_{Re} = 40$. This can be explained by the fact that since the shear rate decreases exponentially with distance from the impeller (*7*), the fluid outside the immediate region of the impeller has a higher apparent viscosity than that near the impeller and therefore tends to dampen any eddy currents caused by the impeller. As the agitation is increased, the shear rate also increases. This decreases the apparent viscosity, aiding eddy formation. The transition to turbulent flow is therefore much sharper than with Newtonian fluids, and the transition region between laminar and turbulent flow is very short.

The curves for Newtonian and pseudoplastic fluids coincide once more at Reynolds numbers greater than 300. As with Newtonian fluids, vortices start to form at a Reynolds number of about 340, at which the curves for baffled and unbaffled vessels begin to diverge. The conventional $N_{Po} - N_{Re}$ correlation for Newtonian fluids therefore differs from that for pseudoplastic fluids only in the early transition region where N_{Re} is between 10 and 300.

In these tests the rotational speeds and shear rates varied over a wide range, and the flow-behavior index varied considerably. The overall change in *n'* for all fluids tested was from 0.28 to 1.00. In tests with single fluids *n'* varied from 0.34 to 0.63, 0.28 to 0.50, 0.57 to 0.98, and so forth.

The results shown in Figure 3 agree with those given by Metzner and co-workers (*4*) for single flat-bladed turbines and various non-Newtonian fluids of essentially constant *n'*. This is further substantiation of the general nature of Equation (2).

BLEND TIMES

In previous studies the time required to create a high degree of uniformity in an agitated liquid system

TABLE 1. RANGE OF VARIABLES STUDIED

Vessel diameter, in.	5.7 to 17.3
Impeller diameter, in.	2 to 6
Reynolds numbers for power measurements	0.6 to 900
Impeller rotational speed, rev./sec.	0.5 to 21
Apparent viscosity, poises	1 to 100
Index *n'*	0.28 to 1.00
Vessel-to-impeller-diameter ratio	3:1
Fluid depth, in.	6 to 18
Ratio of depth to vessel diameter	1:1

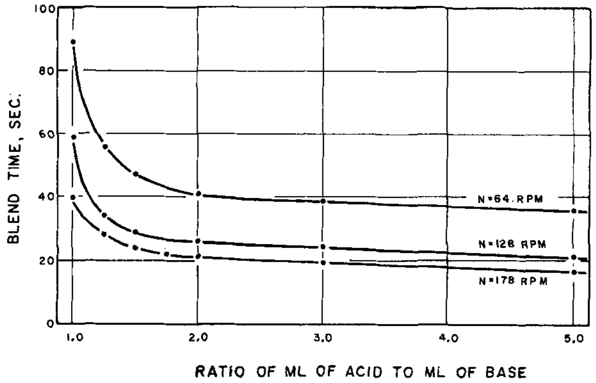

Fig. 4. Effect of acid-to-base ratio on terminal blend time.

has been called the *mixing time.* Mixing however is a general term which refers to a broad category of operations. The operation studied here was the blending of two miscible fluids, and the times required to achieve a specified degree of uniformity were therefore called *blend times.*

The ultimate aim of blending is to create a condition of complete uniformity on a molecular level between the added ingredients and the original volume of liquid. The time required to do this would be the true *blend time.* Experimental techniques however do not always detect this degree of uniformity. The method used by previous workers *(2, 7)* involved visual observation of an indicator color change when a base was neutralized with a stoichiometrically equivalent amount of acid. This is called *terminal blending,* and the time required is called *terminal blend time.*

Not all blending operations are carried to terminal blending, for often it is sufficient to reach 80 or 90% of terminal blending. Also a major part of the blending is rapid, and much of the terminal blend time is required to reach homogeneity on the molecular level.

This is borne out in visual observation of the color changes. The liquid initially is red. When the acid is introduced, no immediate change is observed for a short time, the interval depending upon the overall length of time to attain complete neutralization. Then the solution clears very rapidly until it is colorless except for regions of color in the form of wisps or rings, which then are slowly neutralized. In some cases this final process is so slow that it apparently takes place by diffusion alone. The blend time is divided into two parts: the time required for the overall color change, and the time to neutralize the remaining wisps. The first part might be called the *gross blend time.* The second part is at least 75% of the terminal blend time and in some cases 90% or more, depending upon the flow patterns in the liquid and the ratio of acid to base used.

Terminal blend times were sometimes very long and always hard to reproduce. Identical tests gave times that differed by as much as 20%. To devise a more reproducible measure of blend times a study was made of the effect of acid-to-base ratio on blend time, with the results shown in Figure 4. For a rotational speed of 64 rev./min. the terminal blend time was 89 sec. with an average deviation of 3.6 sec. With a 2:1 ratio (twice the stoichiometric amount of acid present) the blend time was 40.6 sec. with an average deviation of 0.8 sec. With acid-to-base ratios greater than 2:1 the time fell slightly with increasing ratios. As the ratio diminished toward 1:1, the blend time increased exponentially.

Since blend times with a 2:1 ratio were reproducible, and since higher acid-to-base ratios had little effect on blend time, a ratio of 2:1 was used in the blending studies. The blend times found with this ratio are shorter than the terminal blend times, since excess acid is present, and the color change can occur throughout the tank even though parts of the tank contents are not uniform. The blend times observed with excess acid present are called *partial blend times.* With a 2:1 acid-to-base ratio the partial blend times consistently averaged 44% of the terminal blend times. The values reported in this paper are the average of three individual measurements, corrected to terminal blend times by dividing by 0.44.

The fact that blending is accompanied by a color change affords an excellent opportunity for observing the manner in which blending occurs. In unbaffled vessels there are two overall flow patterns that control the blend time. With agitator speeds below that which initiates the formation of a vortex the flow pattern generates two rings located in planes parallel to and just above and below the plane of the impeller and two-thirds of the distance between impeller and vessel wall. This is shown in Figure 5. Flow around the rings extends through the bulk of the fluid in well-defined planes. Laminar flow exists in the region outside the impeller. There is a small region around the impeller where thorough blending occurs. The outer flow channels all pass through this region. Blending is therefore accomplished by two mechanisms: turbulent blending in the region around the impeller and bulk circulation of fluid in laminar flow transporting the blended fluid from the impeller zone to the rest of the vessel. Since the flow lines of the two rings do not extend into the impeller blending zone, flow here remains laminar and blending occurs by diffusion. This was observed at all speeds up to the onset of a vortex. The bulk fluid was neutralized by the acid in 1 or 2 min., but the two rings of unneutralized base remained for 10 min. or more.

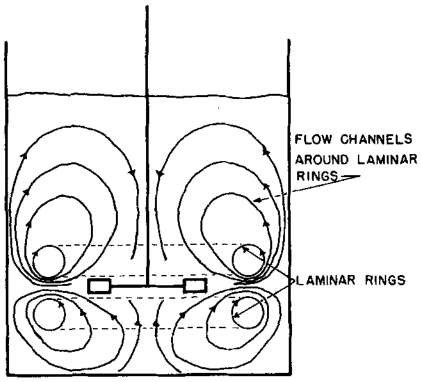

Fig. 5. Flow patterns in an unbaffled vessel agitated with a single flat-bladed turbine.

Blend times were somewhat lengthened, not shortened, with the insertion of baffles. Baffles disturbed the flow patterns so that the rings did not form, but there developed between the baffles regions of laminar flow where blending occurred only by diffusion.

A second type of flow pattern occurs at agitator speeds between that which just causes a vortex to form and the speed which extends the vortex to a point just above the impeller. In this region clearly defined flow patterns do not exist and blending is rapid. Diffusion is no longer controlling.

Inserting baffles again increased the blend time since they caused the formation of stagnant pockets of fluid, even when the baffles were placed ¼ in. away from the wall of the vessel.

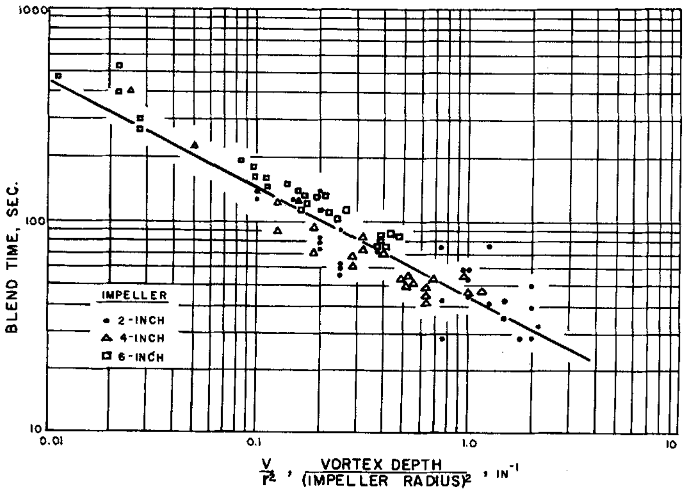

Fig. 6. Correlation of blend time for agitation in unbaffled vessels with vortexing.

These regions of laminar flow extended from points between the baffles to a point well inside the bulk of the fluid. There were no runs in which the addition of baffles decreased the blend time. Even when the use of baffles made possible a substantial increase in impeller speed above that at which an unbaffled vessel could operate without trapping air in the fluid, the blend time with baffles was very long. The effectiveness of blending a pseudoplastic fluid was unequivocally inhibited by inserting radial baffles.

The region of effective operation of a flat-bladed turbine in blending a pseudoplastic fluid therefore extends from the onset of vortex formation to severe vortexing.

In measurement of blend times the approximate depth of the vortex was recorded. Initially this was done merely for curiosity, but is soon became apparent that the blend time was the same for fluids of different viscosities when the vortex was of the same shape. When there is no vortex the blend time is very long, but once a vortex is formed the required time drops suddenly and continues to decrease with increased vortexing. Apparently blending is governed largely by flow patterns, and the flow patterns for geometrically similar systems are similar when the vortices have the same shape.

Vortex depth therefore became the basis for correlating blend time in the various agitated vessels. Blend time is related to vortex depth, as shown in Figure 6, by the relationship

$$t = 45rV^{-1/2} \qquad (3)$$

Equation (3) applies only to unbaffled vessels and agitator speeds which cause vortexing. A rule of thumb for rapid blending is to operate at a rotational speed which produces a vortex.

Fox and Gex (*2*) and Norwood and Metzner (*7*), through a dimensional analysis, developed correlations between blend time and Reynolds number in Newtonian fluids. Norwood and Metzner presented a curve for turbine impellers similar to those used in this study and suggest that it may be applied to blending pseudoplastic fluids, without however stating how the apparent viscosity is to be calculated. For comparison blend times for Natrosol solutions were calculated from the Norwood and Metzner correlation, with an apparent viscosity computed as in the power number–Reynolds number study (Figure 3). The results did not agree with experiment; the measured blend times were ten to fifty times as long as the calculated values. Norwood and Metzner's correlation is for fully baffled vessels, whereas the present data are for unbaffled tanks. However insertion of baffles lengthened the mixing times for Natrosol solutions, increasing rather than reducing the discrepancy.

It would be possible to define an apparent viscosity for blending pseudoplastic fluids in such a way that the data would agree with those for Newtonian fluids. However the data points would then fall, on Norwood and Metzner's plot, in a region of laminar flow. Visual observations indicated that flow behavior could best be described as middle transition—definite turbulence in regions near the impeller and laminar flow near the walls and fluid surface. Blending pseudoplastic fluids therefore takes much longer than blending Newtonian fluids of similar viscosity because of the sharp decrease in fluid movement with increasing distance from the impeller. This decrease was visually observed by Metzner and Taylor (*6*).

CONCLUSIONS

1. For determining the apparent viscosity of a non-Newtonian fluid in an agitated vessel the equation relating average shear rate to impeller speed applies to pseudoplastic fluids with a flow-behavior index that varies with shear rate, as well as to those fluids that have a constant flow-behavior index.
2. Variations of the flow-behavior index for pseudoplastics do not appear to limit the usefulness of the general power number–Reynolds number correlation for non-Newtonian fluids.
3. Blending of pseudoplastic fluids is most rapid in unbaffled vessels with agitator speeds which cause a vortex to form. Blend time is inversely proportional to the square root of the vortex depth.

NOTATION

dv/dr = shear rate, sec.$^{-1}$
D = impeller diameter, ft.
g_c = conversion factor, (ft.) (lb.-mass)/(sec.^{2}) (lb.-force)
K = flow consistency index, (lb.-force) (sec.^{n})/(ft.^{2})
k = proportionality constant in Equation (2), dimensionless
N = rotational speed, rev./sec.
N_{po} = power number, dimensionless, $N_{po} = Pg_c/D^5N^3\rho$
N_{Re} = Reynolds number, dimensionless, $N_{Re} = D^2N\rho/\mu_a$
n' = flow-behavior index, dimensionless
P = power, (ft.) (lb.-force)/sec.
r = radius of impeller, in.
t = terminal blend time, sec.
V = vortex depth, in.
μ_a = apparent viscosity, (lb.-mass)/(ft.) (sec.)
ρ = fluid density, (lb.-mass)/cu. ft.
τ = shear stress, dynes/sq. cm.

LITERATURE CITED

1. Calderbank, P. H., and M. B. Moo-Young, *Trans. Inst. Chem. Engrs.* (*London*), 37, 26 (1959).
2. Fox, E. Q., and V. E. Gex, *A.I.Ch.E. Journal*, 2, 539 (1956).
3. Krieger, I. M., and S. H. Maron, *J. Appl. Phys.*, 25, 72 (1954).
4. Metzner, A. B., R. H. Feehs, H. L. Ramos, R. E. Otto, and J. D. Tuthill, *A.I.Ch.E. Journal*, 7, 3 (1961).
5. Metzner, A. B., and R. E. Otto, *ibid.*, 3, 3 (1957).
6. Metzner, A. B., and J. S. Taylor, *ibid.*, 6, 109 (1960).
7. Norwood, K. W., and A. B. Metzner, *ibid.*, p. 432.
8. Rushton, J. H., E. W. Costich, and H. J. Everett, *Chem. Eng. Progr.*, 46, 395, 467 (1950).

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